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RANDOM PULSATIONS IN A HOMOGENEOUS COARSELY DISPERSED GAS SUSPENSION

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Internal pulsations of both phases of a monodisperse suspension of large particles in a gas are considered under the assumption of statistical independence of particles and isotropy of their pulsations isotropy provided by particle collisions. Statistical characteristics of the pulsations are computed as functions of the mean concentration and physical parameters for macroscopically uniform states of the gas suspension, disregarding the energy dissipation in particle collisions.

Random pulsations of particles and the surrounding liquid in suspensions, fluidized beds, and other disperse systems play a fundamental role in diverse transfer processes occurring in them and in the formation of effective rheological properties of these systems that specify features of the macroscopic motion of their phases and close an appropriate system of conservation equations. Such equations appear essentially different for dispersions of fine and large particles. Below we consider only systems with relatively large identical spherical particles, when their momentum and energy exchange results practically just from direct collisions. The conditions, under which such a situation is realized in dilute dispersions of fine spheres, are considered in [1]; an increase in the particle size and dispersion concentration makes it easier to establish such an interparticle exchange mechanism.

As is shown in [2], hydrodynamic modeling of the disperse system can be carried out in this case in much the same manner as in solving a similar problem for dense gases. The mass and mean momentum equations for continuous and disperse phases and a conservation (or transfer) equation for pulsation energy of the particles that is analogous to the heat conduction equation were derived in [2]. In this case it is possible to obtain explicit representations for stresses due to pulsations and for the pulsation energy flow, so that complete closure of the governing equations requires that only a scalar quantity be determined, namely, the doubled mean pulsation energy per translational degree of freedom of the particle, which is similar in meaning to the temperature in molecular systems with random thermal motion, in a macroscopically uniform state of the disperse system.

The latter is feasible within the framework of an analysis of the pseudoturbulent motion of the particles and the surrounding liquid based on the correlation theory of steady random processes [3]. Here a serious difficulty arises that is related to allowing for the effect of particle collisions on the properties of the pseudoturbulence. This effect was neglected altogether in [2, 4] and the energy flow to the pulsations was estimated as if there were no collisions whatever. Below, this difficulty is surmounted. It appears that ignoring collisions results in highly overestimated quantities characterizing the rate of the particle pulsations.

Below, we examine only suspensions of identical spherical particles in gases whose inertia and weight are negligible compared with similar quantities for the particles. The state of the disperse system is considered to be macroscopically uniform in the sense that its mean characteristics do not depend on the coordinates. (The exception is, of course, the mean liquid pressure, which depends linearly on the vertical coordinate). This state is also assumed to be steady.

Forces Acting on the Particles. Assuming the particles to be fairly large, for the hydraulic force of the interphase interaction we use a quadratic law following from the model of a jet flow past the particles of a concentrated system [5]. For one particle

$$\mathbf{f}_h = \frac{d_0}{a}\frac{\rho}{n}K(\rho)u\mathbf{u},\quad K = \frac{3\zeta}{8}\left(\frac{1-\rho}{1-1{,}17\rho^{2/3}}\right)^2,\quad \mathbf{u} = \mathbf{v}-\mathbf{w}, \tag{1}$$

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here the resistance coefficient for large spheres is equal to about 0.5. For the buoyancy acting on one particle, we take, unlike [2],

$$\mathbf{f}_b = -(\rho/n)\, d\mathbf{g}, \quad d = \rho d_1, \tag{2}$$

where the gas density is disregarded in accordance with the assumptions made.

Equations (1) and (2) refer to systems with a fixed arrangement of the particles. In real media with pulsating particles, all quantities characterizing a local instantaneous state represent the sums of their mean values and fluctuations relative to them. For simplicity, the angular brackets in notation for the mean concentrations, particle and liquid velocities, as well as liquid pressure in the interparticle spacings are henceforth omitted.

Regarding the pulsations as relatively weak, from Eqs. (1) and (2) we obtain for the mean forces

$$\langle \mathbf{f}_h \rangle = \frac{d_0}{a}\frac{\rho}{n}\left\{ K\left[u\mathbf{u} + \langle (\mathbf{u}_0\mathbf{u}')\mathbf{u}' \rangle\right] + \frac{dK}{d\rho}\left[u\langle \rho'\mathbf{u}' \rangle + \right.\right.$$

$$\left.\left. + \langle \rho'(\mathbf{u}_0\mathbf{u}') \rangle \mathbf{u}\right] + \frac{1}{2}\frac{d^2K}{d\rho^2}\langle \rho'^2 \rangle u\mathbf{u} \right\}, \quad \mathbf{u}_0 = \frac{\mathbf{u}}{u}, \tag{3}$$

$$\langle \mathbf{f}_b \rangle = -(\rho/n)\, \rho d_1 \mathbf{g},$$

and for their fluctuations

$$\mathbf{f}_h' = \frac{d_0}{a}\frac{\rho}{n}\left\{ K\left[u\mathbf{u}' + (u_0\mathbf{u}')\mathbf{u}\right] + \frac{dK}{d\rho}\rho' u\mathbf{u} \right\},$$

$$\mathbf{f}_b' = -(\rho/n)\, \rho' d_1 \mathbf{g}. \tag{4}$$

In Eqs. (3) and (4), K and its derivatives are calculated, of course, for the mean volume concentration.

The Langevin equation for a particle executing a random pulsatory motion is written as

$$m(\partial \mathbf{w}'/\partial t) = \mathbf{f}_h' + \mathbf{f}_b' + \mathbf{f}_c, \tag{5}$$

where $\mathbf{f}_c$ is the random force acting on a particle in a collision with other particles. In Eq. (5) a coordinate system where the mean velocity w of the dispersed phase is zero issued and the relative smallness of $\mathbf{w}'$ is again assumed.

The force $\mathbf{f}_c$, regarded as a function of time, is zero everywhere, except during random time intervals, over which the particle experiences collisions. (For ideally rigid particles, such intervals contract to zero and $\mathbf{f}_c$ represents a sum of delta functions). Of interest, of course is not this random process, resembling a Poisson random process, but rather the expression for the force, averaged over time intervals greatly exceeding the mean time between collisions. This suggests averaging over the assembly of possible situations in the vicinity of an isolated sphere having the random velocity $\mathbf{w}'$, which differ as regards both the positions and the velocities of the surrounding particles. Below all fluctuating quantities are understood as assembly-mean in the sense indicated above.

After averaging over that set, $\mathbf{f}_c$ becomes a continuous function of time. This function can contain constituents proportional only to two vectors, specifically, to the velocity $\mathbf{w}'$ and the vector $\mathbf{u}_0$ that defines the only distinguished direction, characteristic of the average state of the disperse medium. Also, $\mathbf{f}_c$ obviously should depend linearly on $\mathbf{w}'$. Thus, in the most general case it is possible to assume that

$$\mathbf{f}_c = -d_1(\rho/n)\left[A\mathbf{w}' + B(\mathbf{u}_0\mathbf{w}')\mathbf{u}_0\right], \tag{6}$$

where A and B are certain coefficients that are so far unknown. The mean with respect to Eq. (6) goes identically to zero. This implies that the role of the collision force reduces solely to energy redistribution between the degree of freedom in the direction $\mathbf{u}_0$ and the degrees of freedom in transverse directions that are not tantamount to the first one. Furthermore, in the general case the mean $\langle \mathbf{f}_c\mathbf{w}' \rangle$, characterizing dissipation of the kinetic energy of the particles in collisions, differs from zero. In the current study, such dissipation is disregarded for the sake of simplicity. Then A and B should be determined from the conditions of statistical isotropy of the particle pulsations and of zero mean work of the collision force on random migrations of the particles.

In accordance with the discussion in [2], we totally neglect the excitation of rotational degrees of freedom of the particles due to their interaction with the gas and to collisions.

Equation for Pulsatory and Average Motion. Multiplying Eq. (5) by the numerical concentration of the particles yields an equation playing the role of the momentum equation for the pulsatory motion of the disperse phase

$$d_1\rho\,(\partial \mathbf{w}'/\partial t) = n\,(\mathbf{f}_h' + \mathbf{f}_b') + n\mathbf{f}_c. \tag{7}$$

This equation should be completed with the equations resulting from the laws of mass and momentum conservation for the gas phase. The latter were treated in [2], and they can be written as

$$(\partial/\partial t + \mathbf{u}\nabla)\rho' = \varepsilon\nabla\mathbf{v}',\quad 0 = -\nabla p' - n\,(\mathbf{f}_h' + \mathbf{f}_b'),\quad \varepsilon = 1 - \rho. \tag{8}$$

Here a coordinate system with $\mathbf{w} = 0$ and the relative smallness of pulsations were also used.

It is convenient to employ, as in [2, 4], the mathematical apparatus of the correlation theory of steady random processes [3], according to which any random quantity with zero mean is represented as a stochastic Fourier-Stieltjes integral with a random measure, the mean of the square of whose modulus defines the spectral density, for example:

$$\rho'(t, \mathbf{x}) = \int e^{i\omega t + i\mathbf{k}\mathbf{x}}\, dZ_\rho,\quad \Psi_{\rho,\rho}(\omega, \mathbf{k}) = \lim_{d\omega\to 0,\, d\mathbf{k}\to 0} \frac{\langle dZ_\rho dZ_\rho\rangle}{d\omega d\mathbf{k}}$$

Then, the correlation functions can be found by integration with respect to frequency and over the wave space, in particular:

$$\langle \rho'(t, \mathbf{x})\,\rho'(t+\tau, \mathbf{x}+\mathbf{r})\rangle = \int_{-\infty}^{\infty} d\omega \int d\mathbf{k}\, e^{i\omega\tau + i\mathbf{k}\mathbf{r}}\,\Psi_{\rho,\rho}(\omega, \mathbf{k}).$$

Substituting the expressions for all random fluctuations in terms of Fourier-Stieltjes integrals into Eqs. (7) and (8), we arrive at a system of linear algebraic equations for random measures that allow all of them to be expressed in terms of dZ_ρ. After simple manipulations we have

$$\begin{gathered}(\omega + \mathbf{u}\mathbf{k})\, dZ_\rho = \varepsilon\mathbf{k}d\mathbf{Z}_v,\\ (i\omega + A)\, d\mathbf{Z}_w + B\,(\mathbf{u}_0 d\mathbf{Z}_w)\,\mathbf{u}_0 = -i\mathbf{k}dZ_\pi,\\ 0 = -i\mathbf{k}dZ_\pi - F\,[u d\mathbf{Z}_u + (\mathbf{u}_0 d\mathbf{Z}_u)\,\mathbf{u}] - [(dF/d\rho)\,u\mathbf{u} -\\ - \mathbf{g}]\, dZ_\rho,\quad F = K/\varkappa a,\quad \varkappa = d_1/d_0,\quad dZ_\pi = dZ_p/\rho d_1.\end{gathered} \tag{9}$$

Here expressions (3), (4), and (6) for random forces are used.

By solving these equations it is easy to subsequently obtain the expressions for all spectral densities as quantities proportional to $\Psi_{\rho,\rho}(\omega, \mathbf{k})$, and thereafter to compute the correlation functions needed. Closing these expressions requires wing the spectral density for concentration fluctuations that was found in [6]. Below, the following formulas will be needed:

$$\int_{-\infty}^{\infty} \Psi_{\rho,\rho}(\omega, \mathbf{k})\, d\omega = \Phi(k),\quad \int \Phi(k) k^2 dk = \frac{\langle \rho'^2\rangle}{4\pi}. \tag{10}$$

For the dispersion of the random concentration field we will utilize an expression following from the Perkus-Yevik theory of dense gases and liquids that was also obtained in [6]:

$$\langle \rho'^2\rangle = \rho^2\left[1 + 2\rho\,\frac{4-\rho}{(1-\rho)^4}\right]^{-1}. \tag{11}$$

The mean parameters, characterizing a macroscopically uniform state of the disperse medium considered, are related by the equations

$$-\nabla p - n(\langle \mathbf{f}_h\rangle + \langle \mathbf{f}_b\rangle) = 0,\quad n(\langle \mathbf{f}_h\rangle + \langle \mathbf{f}_b\rangle) + \rho^2 d_1\mathbf{g} = 0, \tag{12}$$

representing a special case of the equations in [2]. The first of relations (12) defines the constant mean pressure gradient of the gas in the homogeneous disperse medium in the gravitational field, and the second connects the mean sliding velocity u with the mean volume concentration of the medium ρ. Setting

$$n\langle \mathbf{f}_h \rangle = (d_0/a)\,\rho\lambda K u \mathbf{u},\quad \mathbf{g} = -g\mathbf{u}_0, \tag{13}$$

from Eqs. (12) we obtain

$$u = \left(\varepsilon \frac{\varkappa a g}{\lambda K}\right)^{1/2} = \left(\frac{\varepsilon g}{\lambda F}\right)^{1/2}, \tag{14}$$

where K, F, and κ are determined, respectively, by Eqs. (1) and (9), and λ is as yet unknown. If the effect exerted on the hydraulic resistance of the particle by quantities quadratic in the fluctuations is disregarded, then Eq. (3) for the mean hydraulic force and Eq. (1) for the force in the system with no fluctuations coincide, and $\lambda = 1$.

We would like to emphasize that the use of Eqs. (10) and (11) corresponds to the most detailed description of fluctuations that is compatible with a continuum approach to the problem [6]. In this case, w' has the meaning of the velocity fluctuation of a single particle, and v' and p', of the gas velocity and pressure disturbances in its specific volume.

Solution of the Equation for Fluctuations and Its Closure. We take the first coordinate axis to run along $\mathbf{u}_0$, and the two others, in an arbitrary way, in the plane normal to the distinguished direction. Then, after cumbersome calculations we obtain from Eq. (9)

$$\begin{gathered}
dZ_v = dZ_w + dZ_u,\quad dZ_{u2,3} = -i\,(k_{2,3}/Fu)\,dZ_\pi,\\
dZ_{u1} = -i\,(k_1/2Fu)\,dZ_\pi - (M - 1/\varepsilon)\,u\,dZ_\rho,\\
dZ_{w2,3} = -\frac{ik_{2,3}}{i\omega + A}\,dZ_\pi,\quad dZ_{w1} = -\frac{ik_1}{i\omega + A + B}\,dZ_\pi,\\
dZ_\pi = ik^{-2}(\omega/\varepsilon + Muk_1)\,[i\omega(2A + B) + A(A + B) - \omega^2] \times\\
\times \{i\omega\,[2Fu + (2A + B)(2 - t^2)] + 2Fu\,[A + B(1 - t^2)] +\\
+ (2 - t^2)\,[A(A + B) - \omega^2]\}^{-1} 2Fu\,dZ_\rho,
\end{gathered} \tag{15}$$

where the designations (we use Eqs. (9), (13), and (14))

$$M = \frac{1}{\varepsilon} + \frac{1}{2Fu^2}\left(\frac{dF}{d\rho}u^2 + g\right) = \frac{1}{2}\left(\frac{d\ln K}{d\rho} + \frac{2+\lambda}{\varepsilon}\right),\quad t = \frac{k_1}{k}. \tag{16}$$

are introduced.

The calculations can be simplified considerably, if it is noticed that the characteristic frequency, defining the evolution of concentration fluctuations, is of the order of magnitude of $k_0^2 D \sim (a^2/\rho^{2/3})\,D$, where D is the coefficient of pseudoturbulent self-diffusion of the particles [6]. On the other hand, it is seen from Eqs. (15) that A and B should be of the order of magnitude of Fu, having the dimension of frequency. It is not difficult to show using the results from [6] and the above definition of Fu that the first quantity is generally much smaller than the second. This implies that the terms in 3 in Eq. (15) can approximately be ignored, as compared with the terms involving Fu, A, and B, i.e., the fluctuations can be considered in a quasi-steady approximation. Such an approximation corresponds to actually neglecting the change in the random concentration field over times of the order of the temporal scale of velocity fluctuations.

Introducing the new unknown parameters

$$x = A/2Fu,\quad y = B/2Fu, \tag{17}$$

results in the following approximate relations for random measures replacing those in Eq. (15):

$$dZ_{u2,3} = 2x\,dZ_{w2,3},\quad dZ_{u1} = (x + y)\,dZ_{w1} - (M - 1/\varepsilon)\,u\,dZ_\rho,$$

$$dZ_{w2,3} = \frac{(x+y)\,t\,(k_{2,3}/k)}{L\,(b-t^2)} MudZ_\rho,$$
$$dZ_{w1} = \frac{xt^2}{L\,(b-t^2)} MudZ_\rho, \quad L = y + x\,(x+y), \tag{18}$$
$$b = L^{-1}[x + y + 2x\,(x+y)],$$

here, M as a function of ρ is determined by Eq. (16). First of all, let us find representations for the unknown parameters in Eq. (17).

Based on Eq. (18), the condition of statistical isotropy of pseudoturbulent particle pulsations $\langle w_1'^2 \rangle = \langle w_2'^2 \rangle$ may be written as

$$\frac{x^2}{L^2}\left(\int_0^1 \frac{t^4 dt}{(b-t^2)^2}\right)(Mu)^2 \langle \rho'^2 \rangle =$$
$$= \frac{(x+y)^2}{2L^2}\left(\int_0^1 \frac{t^2(1-t^2)}{(b-t^2)^2}\,dt\right)(Mu)^2 \langle \rho'^2 \rangle .$$

Hence the equation

$$x^2 \int_0^1 \frac{t^4}{(b-t^2)^2}\,dt = \frac{(x+y)^2}{2}\int_0^1 \frac{t^2(1-t^2)}{(b-t^2)^2}\,dt.$$

follows. The second equation for x and y follows from the requirement that the work of the collision force go to zero, which gives

$$\langle f_c w' \rangle = -d_1(\rho/n)\,(A\langle w'^2\rangle + B\langle w_1'^2\rangle) \sim 3x + y = 0.$$

Using this to express y in terms of x, we arrive at a unique solution for x. The analysis indicates that a physically acceptable solution for this equation conforms to $b = s^2 > 1$ and is unique. In this case the equation itself reduces to the form

$$\frac{s(s^2-1)}{2}\ln\frac{s+1}{s-1} = \frac{3s^2-2}{9s^2-2} + s^2 - 1.$$

A numerical solution of this equation determines s, after which x and y may easily be found. We obtain

$$s = 1{,}598, \quad x = -5{,}122, \quad y = 15{,}366. \tag{19}$$

This immediately defines the mean square of any component of the particle velocity fluctuation and the effective "temperature" of the pseudoturbulent motion in the form

$$\langle w_i'^2 \rangle = C_{w,w} M^2 \langle \rho'^2 \rangle u^2 = W(\rho)\,(u^\circ)^2,$$
$$\theta_e = m\,\langle w_i'^2 \rangle = W(\rho)\,m\,(u^\circ)^2, \quad C_{w,w} = 1{,}17\cdot 10^{-3}, \tag{20}$$

where the function $W(\rho)$ and the velocity u^0 of the descent of a single particle in an unbounded gas are introduced:

$$W(\rho) = C_{w,w}\frac{(1-1{,}17\rho^{2/3})^2}{(1-\rho)\,\lambda} M^2(\rho)\,\langle \rho'^2 \rangle, \quad u^\circ = \left(\frac{8\varkappa}{3\xi}\right)^{1/2}(ag)^{1/2}, \tag{21}$$

and M and $\langle \rho'^2 \rangle$ are determined by Eqs. (16) and (11), respectively.

Figure 1a plots the dimensionless temperature $\theta_e/m(u^0)^2$ versus ρ. It follows from a comparison with the results of [2, 4] that the latter results appear to be highly (by more than an order of magnitude) overestimated. There are two reasons for this. First, the allowance for the collision force noticeably changes the phase relationships between w' and fluctuations of other variables in comparison with a collisionless model, which leads to an overestimated result for the flow of energy to the pulsatory motion. Second, the dispersion of the concentration fluctuations (11) is much less than the dispersion used in [2, 4] following from a fairly coarse grid model [7]. Furthermore, a temperature

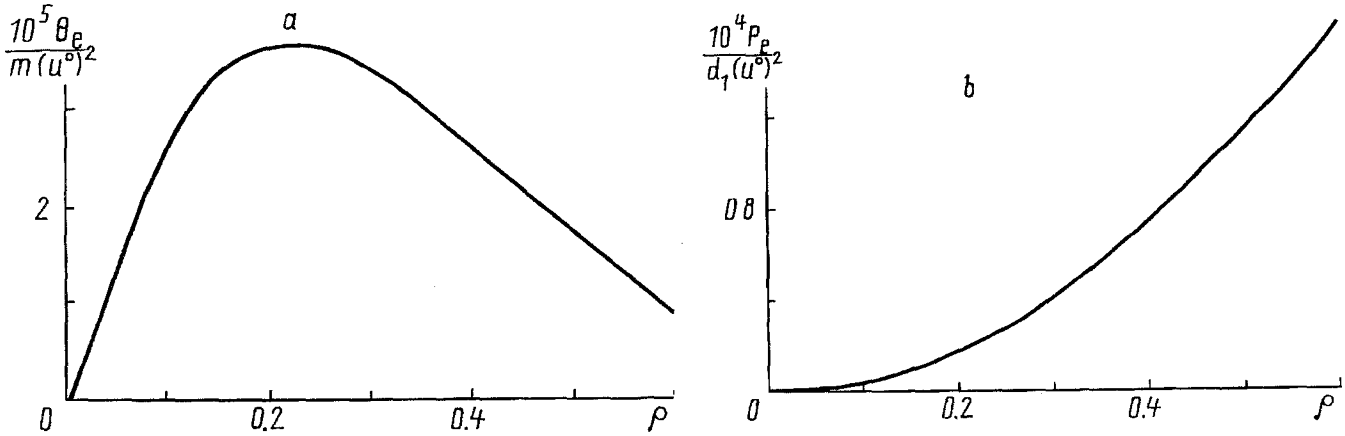

Fig. 1. Dimensionless equilibrium temperature (a) and dimensionless pressure of a pseudogas of particles (b) at $\lambda = 1$.

maximum is attained at a value of ρ only slightly larger than 0.20, which is markedly smaller than the value that would be expected from previous calculations.

Of fundamental importance for the mechanics of coarsely dispersed gas suspensions is the pressure of the pseudogas of suspended particles (or the compressibility modulus of the disperse phase). Based on [2, 6], this pressure in macroscopically uniform states may be expressed as

$$P_e = mn \langle w_j'^2 \rangle G(\rho) = \rho G(\rho) W(\rho) d_1 (u^\circ)^2, \quad G = \frac{1+\rho+\rho^2-\rho^3}{(1-\rho)^3}. \tag{22}$$

Figure 1b plots the dimensionless pressure as a function of ρ. It is of great significance that the dimensionless pressure rises monotonically with increasing mean concentration, which is the condition for thermodynamic stability of the uniform state of the disperse medium [2]. In principle, in calculations use might also be made of other expressions for $\langle \rho'^2 \rangle$ and G, following from other approximate theories of statistical physics, instead of those presented in Eqs. (11) and (22). For example, in this connection Enskog's theory of dense gases was considered in [6]. However, as was pointed out in [6], the latter theory erroneously undervalues the dispersion of concentration fluctuations in concentrated suspensions. Therefore, it is not surprising that the relevant concentration dependence of the effective pressure has a maximum, which is, of course, an artefact. Hence it follows that classical results of the statistical physics of molecular systems should be extended to disperse media with certain caution.

Statistical Characteristics of Pseudoturbulence. Other mean values, characterizing pseudoturbulent motion, can be computed quite analogously. Here we present the expressions for the mean values of quadratic in thefluctuations. Calculations give

$$\begin{gathered}
\langle \rho' w_1' \rangle = C_{\rho,w} M \langle \rho'^2 \rangle u, \quad \langle \rho' w_j' \rangle = 0, \quad j \neq 1; \\
\langle \rho' v_1' \rangle = C_{\rho,v} M \langle \rho'^2 \rangle u - (M - 1/\varepsilon) \langle \rho'^2 \rangle u, \quad \langle \rho' v_j' \rangle = 0, \quad j \neq 1; \\
\langle v_1'^2 \rangle = [C_{v,v}^{(1)} M^2 - E_{v,v}^{(1)} M (M - 1/\varepsilon) + (M - 1/\varepsilon)^2] \langle \rho'^2 \rangle u^2, \\
\langle v_2'^2 \rangle = \langle v_3'^2 \rangle = C_{v,v}^{(2)} M^2 \langle \rho'^2 \rangle u^2; \\
\langle v_1' w_1' \rangle = [C_{v,w}^{(1)} M^2 - E_{v,w}^{(1)} M (M - 1/\varepsilon)] \langle \rho'^2 \rangle u^2, \\
\langle v_2' w_2' \rangle = \langle v_3' w_3' \rangle = C_{v,w}^{(2)} M^2 \langle \rho'^2 \rangle u^2; \\
\langle v_j' v_k' \rangle = \langle v_j' w_k' \rangle = 0, \quad \langle w_j' w_k' \rangle = 0, \quad j \neq k,
\end{gathered} \tag{23}$$

where the following numerical coefficients are introduced:

$$\begin{gathered}
C_{\rho,w} = 2{,}40 \cdot 10^{-2}, \quad C_{\rho,v} = 0{,}270, \quad C_{vv}^{(1)} = 0{,}148, \quad C_{v,v}^{(2)} = 0{,}202, \\
C_{v,w}^{(1)} = 1{,}32 \cdot 10^{-2}, \quad C_{v,w}^{(2)} = -2{,}18 \cdot 10^{-2}, \quad E_{v,v}^{(1)} = 0{,}540, \quad E_{v,w}^{(1)} = 2{,}40 \cdot 10^{-2}.
\end{gathered} \tag{24}$$

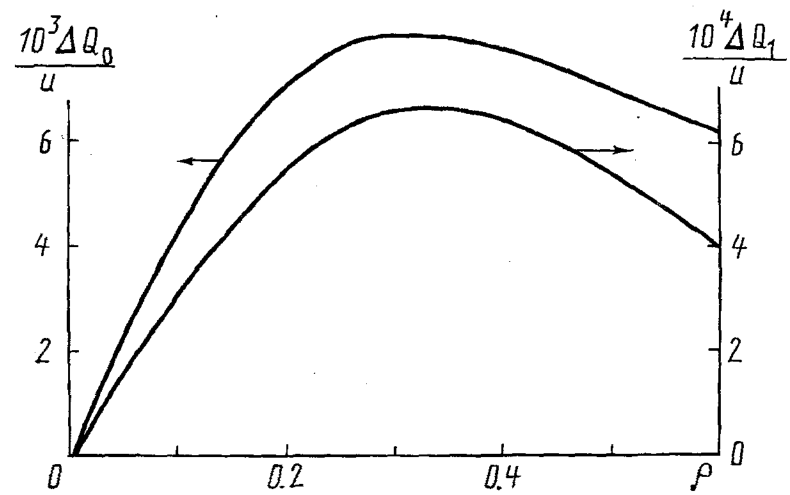

Fig. 2. Relative volume phase flows, resulting from pulsations, at $\lambda = 1$.

It follows from Eqs. (23) and (24) that the gas pulsations remain anisotropic despite isotropy of the particle pulsations and that the fluctuating velocities of the liquid greatly exceed those for the particles. When needed, various double-point double-time correlation functions can also be computed using the expression for the total spectral density of concentration fluctuations from [6].

Effect of Pulsations on the Average State. Pulsations of both phases involve, primarily, a difference between averaged characteristics of the real disperse medium and those of the same medium with fixed particles under identical external conditions. (This is evident if only by comparing the mean force from Eq. (3) with the quantity in Eq. (1).)

The mean volume flows of the phases can be written in the form

$$Q_0 = \langle (\varepsilon + \varepsilon')(v_1 + v_1') \rangle = \varepsilon v_1 + \Delta Q_0, \quad \Delta Q_0 = -\langle \rho' v_1' \rangle,$$
$$Q_1 = \langle (\rho + \rho')(w_1 + w_1') \rangle = \rho w_1 + \Delta Q_1, \quad \Delta Q_1 = \langle \rho' w_1' \rangle. \tag{25}$$

Figure 2 illustrates dimensionless additions to the flows, computable from Eqs. (23)-(25). These additions, resulting from pulsations, lead, in particular, to the fact that experiments on determining the rate of constrained deposition of particles based on measuring the volume or mass phase flows (for example, the rate of deposit buildup on the bottom of a vessel containing the disperse mixture) and on directly tracking the trajectories of individual particles produce different results. However, judging from the curves in Fig. 2, this difference proves to be rather slight for coarsely dispersed mixtures. Yet this is of fundamental importance.

Now we consider a change in the effective hydraulic resistance of the system of suspended particles toward the relative gas flow, caused by the pseudoturbulent motion. To this end, we transform the first of Eqs. (3) using Eqs. (23) and (24) and compare the result with Eq. (13). This leads to an equation for λ, which was introduced in Eq. (13):

$$c_2\lambda^2 + c_1\lambda + c_0 = 0, \quad c_0 = 1 + (0{,}123h_1^2 - 0{,}246h_1h_3 - 0{,}504h_3^2 +$$
$$+ 0{,}492h_3/\varepsilon + h_4)\langle \rho'^2 \rangle, \quad c_1 = -1 + (0{,}123h_1 - 0{,}246h_2 - 0{,}254h_3)\langle \rho'^2 \rangle/\varepsilon,$$
$$c_2 = 0{,}158\langle \rho'^2 \rangle/\varepsilon^2, \quad h_1 = 1/\varepsilon + h_3/2, \quad h_2 = 1/\varepsilon + h_3,$$
$$h_3 = d\ln K/d\rho, \quad h_4 = (1/2K)(d^2K/d\rho^2), \tag{26}$$

where K and $\langle \rho'^2 \rangle$ are defined by Eqs. (1) and (11).

Figure 3 gives the dependence on ρ for the solution of Eq. (26) that reduces to unity at $\rho = 0$. With low concentrations, $\lambda < 1$, which may be perceived as a certain decrease in the hydraulic resistance of the collection of pulsating particles as compared with a similar system of fixed particles. With high concentrations, the opposite is the case.

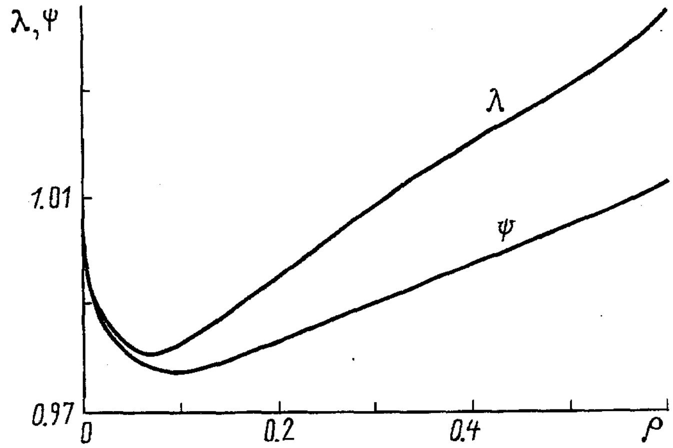

Fig. 3. Concentration dependences of the coefficients of hydraulic resistance variation λ and ψ.

Since λ differs weakly from unity throughout the range of ρ, various mean characteristics of the pseudoturbulence can be computed at $\lambda = 1$, which is done in Figs. 1 and 2. In experiments, instead of the velocity u the relative volume phase flow or the effective velocity u_e, which is determined from the equation Q_0-Q_1 = εu_e, is generally used, and the hydraulic resistance is represented in the form

$$n \langle f_h \rangle = (d_0/a) K_e u_e u_e. \tag{27}$$

In the coordinate system where the mean volume flow of the disperse phase goes to zero, we have $w_1 = \Delta Q_1/\rho$ and $v_1 = u$ (see Eq. (25)), and the expression for ΔQ_0 follows from Eqs. (23) and (24). Therefore,

$$u_e = u \{1 + \varepsilon^{-1} [(M - 1/\varepsilon) - 0{,}270M] \langle \rho'^2 \rangle \}. \tag{28}$$

From a comparison of Eq. (13) with Eq. (27) with consideration of Eq. (28)

$$K_e = \psi K, \quad \psi = \lambda \{1 + \varepsilon^{-1} [(M - 1/\varepsilon) - 0{,}270M] \langle \rho'^2 \rangle \}^{-2}. \tag{29}$$

follows. The dependence $\psi(\rho)$ is also illustrated in Fig. 3.

Thus, the particle pulsations are responsible for a certain difference of the function $K(\rho)$, which must be treated theoretically, from $K_e(\rho)$ determined experimentally. A similar circumstance takes place also for finely dispersed suspensions [8] when the above-mentioned effect is far more noticeable.

The differences between flows and the resistance in media containing relatively immobile and pulsating particles necessitate, generally speaking, a refinement of the hydrodynamic equations for disperse media that were written in [2] by analogy with dense gases when the presence of a continuous medium in the interparticle spacings is entirely inessential. Because these differences are insignificant, there is no need for such a refinement in the first approximation, which allows these equations to be left as they are [2].

NOTATION

a, particle radius; d_0, d_i, densities of the gas and the particle material; f_b, f_c, f_n, buoyancy, collision, and hydraulic forces, respectively; g, acceleration due to gravity; k, wave number; L, parameter from Eq. (18); m, particle mass; n, numerical concentration of the particles; P_e, p, pressures of the particle pseudogas and the gas in spacings between them; Q_0, Q_1, volume flows of the gas and the particles; $s = \sqrt{b}$ for $b > 0$; u, relative velocity of the gas; $u_0 = u/u$; u^o, velocity of descent of a single particle in the unbounded gas; v, w, gas and particle velocities; dZ, random measure; ε, porosity; ζ, resistance coefficient of a particle; θ_e, temperature of the particle pseudogas; $\kappa = d_1/d_0$; λ, ψ, coefficients of variation of resistance ; ρ, volume concentration of the particles; Ψ, Φ, spectral densities; ω, frequency; a prime denotes fluctuations, and angular brackets denote averaging.

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